More AC Analysis

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Impedance and Admittance

Impedance is voltage/current

$$\mathbf{Z} = R + jX$$

R = resistance = Re(*Z*)
X = reactance = Im(*Z*)

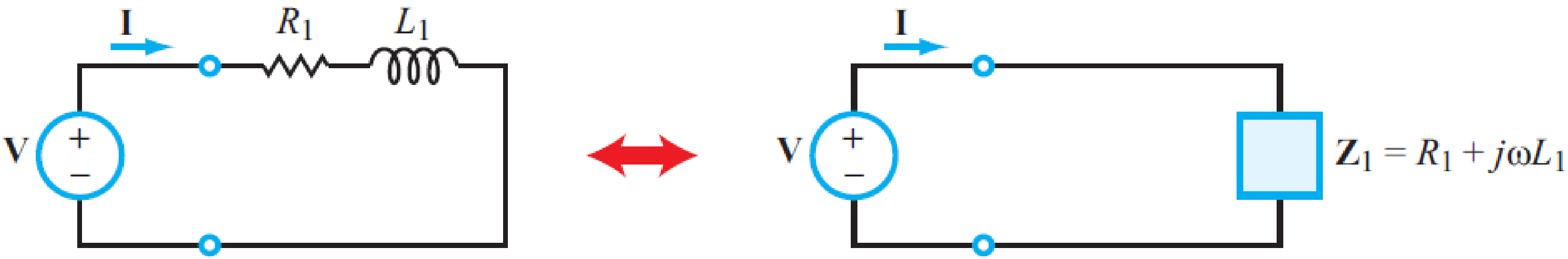
Admittance is current/voltage

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = G + jB$$

G = conductance = Re(*Y*)
B = susceptance = Im(*Y*)

Resistor	$\mathbf{Z} = R$	$\mathbf{Y} = 1/R$
Inductor	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = 1/j\omega L$
Capacitor	$\mathbf{Z} = 1/j\omega C$	$\mathbf{Y} = j\omega C$

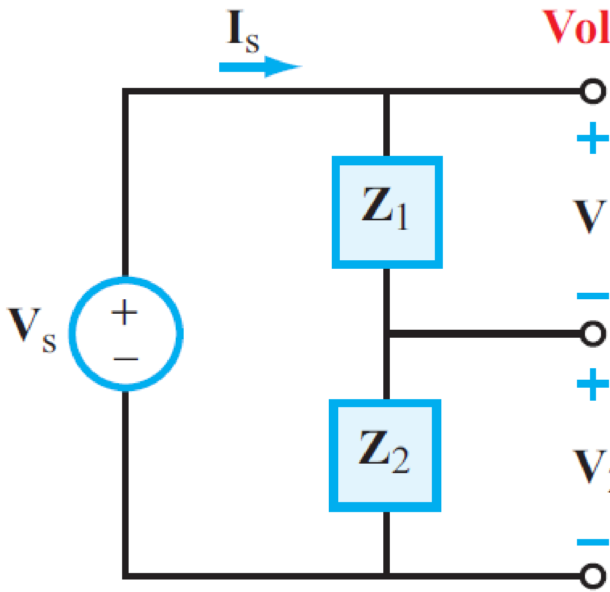
Impedance Transformation



(a) RL

Voltage & Current Division

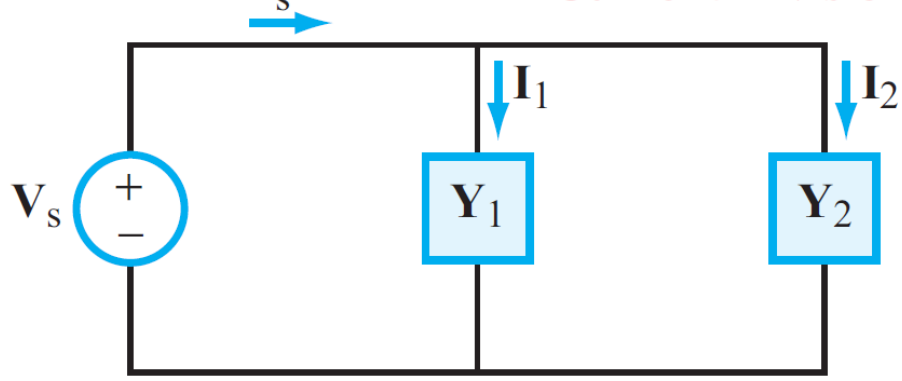
Voltage Division



$$V_1 = \left(\frac{Z_1}{Z_1 + Z_2}\right) V_s$$

$$V_2 = \left(\frac{Z_2}{Z_1 + Z_2}\right) V_s$$

Current Division

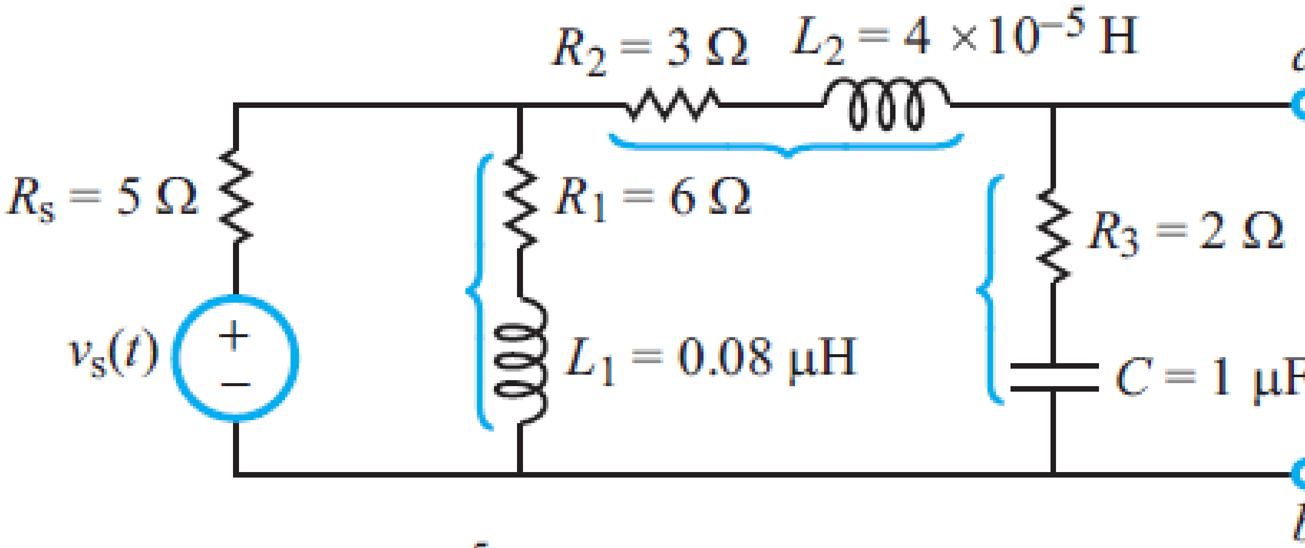


$$I_1 = \left(\frac{Y_1}{Y_1 + Y_2}\right) I_s \qquad I_2 = \left(\frac{Y_2}{Y_1 + Y_2}\right) I_s$$

Linear circuit techniques

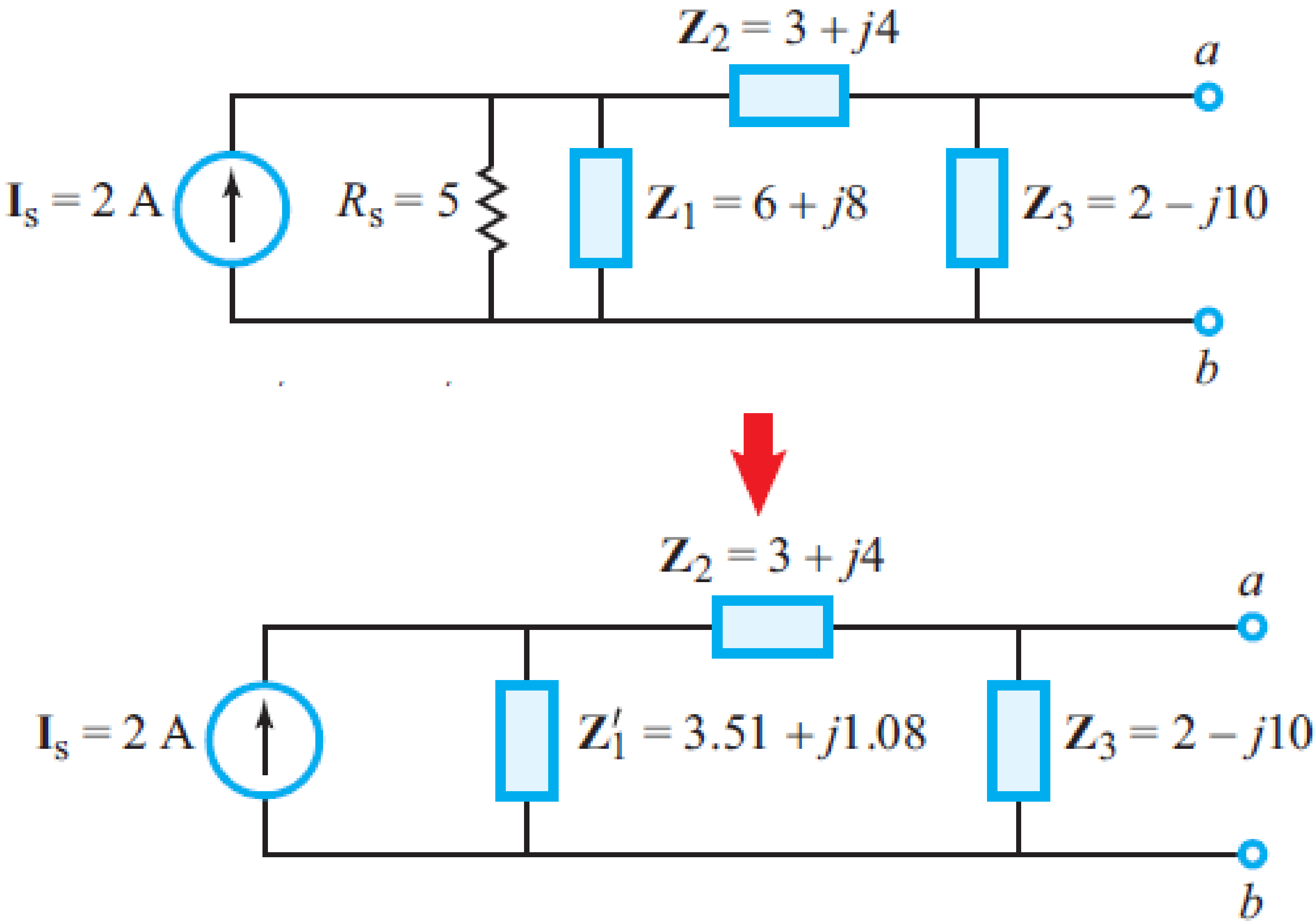
- We can now apply all the techniques we learned before (for dc circuits in the time domain) to ac circuits in the phase domain:
  - Superposition
  - Thevenin / Norton Equivalents

Example: Thévenin Circuit



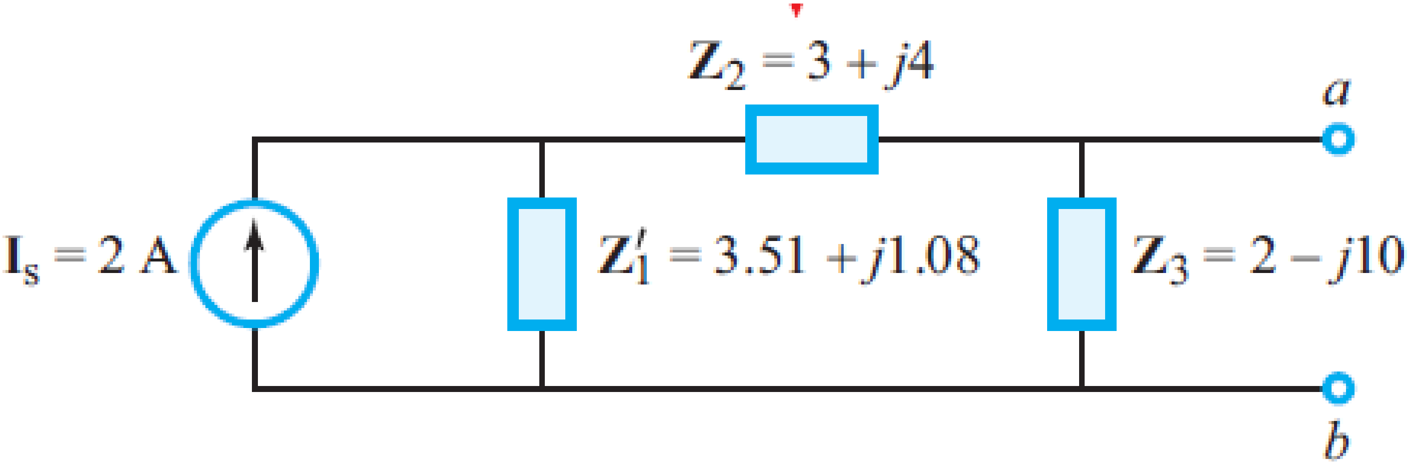
(a) $v_s(t) = 10 \cos 10^5 t$ (V)

Example: Thévenin Circuit

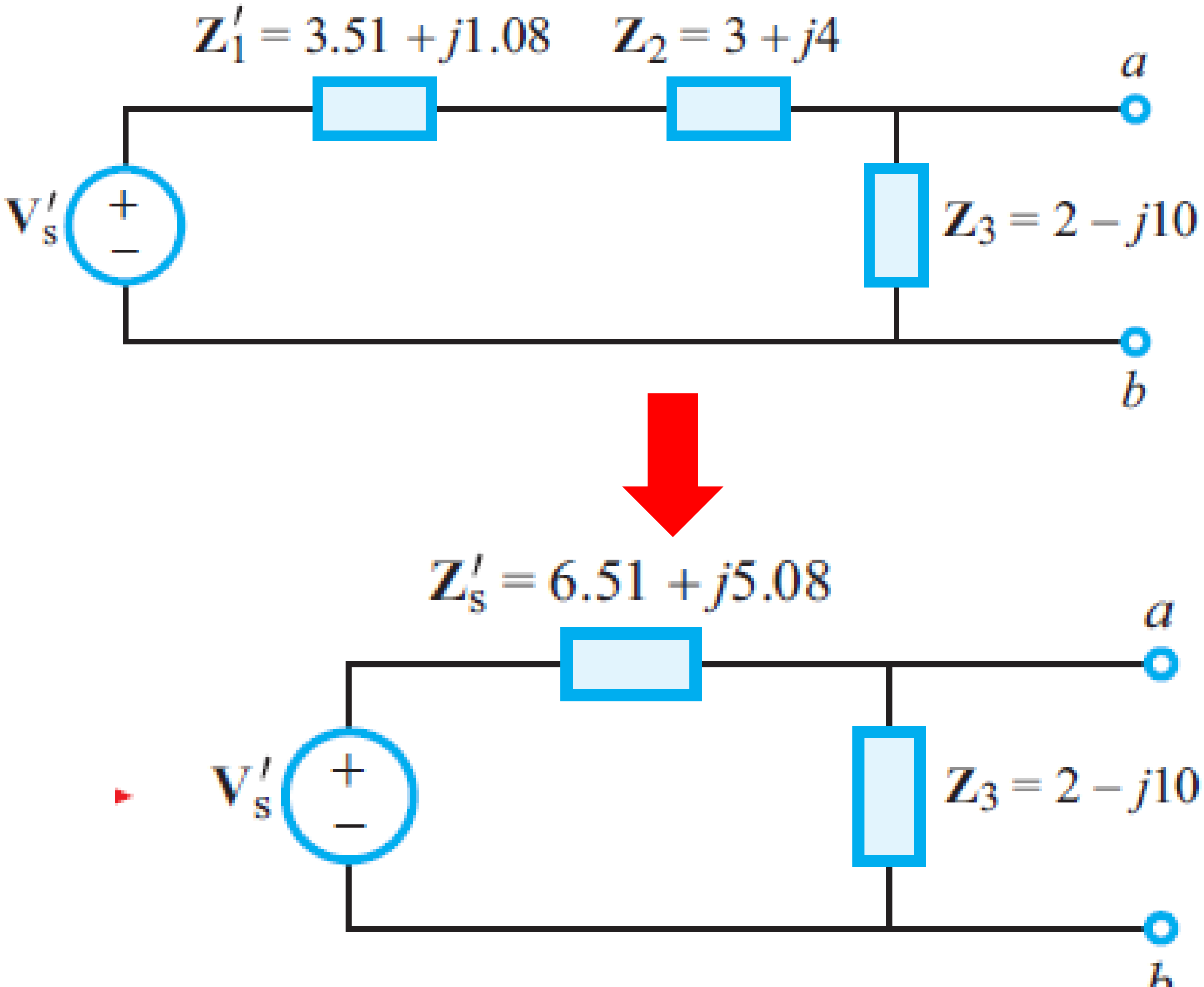


(c) $\mathbf{Z}'_1 = R_s \parallel \mathbf{Z}_1$

Example: Thévenin Circuit

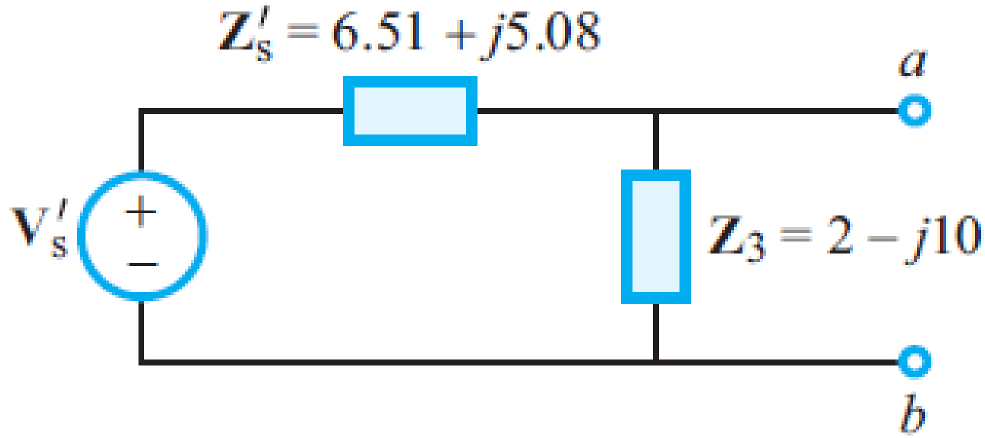


Example: Thévenin Circuit



(e) $Z'_s = Z'_1 + Z_2$

Example: Thévenin Circuit



$$Z_{Th} = Z'_s \parallel Z_3$$

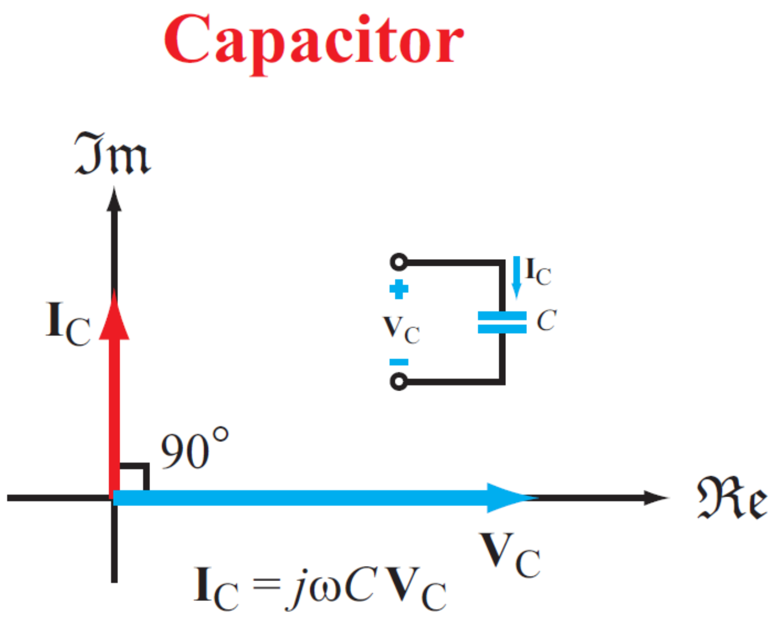
$$\frac{(6.51 + j5.08)(2 - j10)}{(6.51 + j5.08) + (2 - j10)} = (8.42 - j1.59)\ \Omega$$

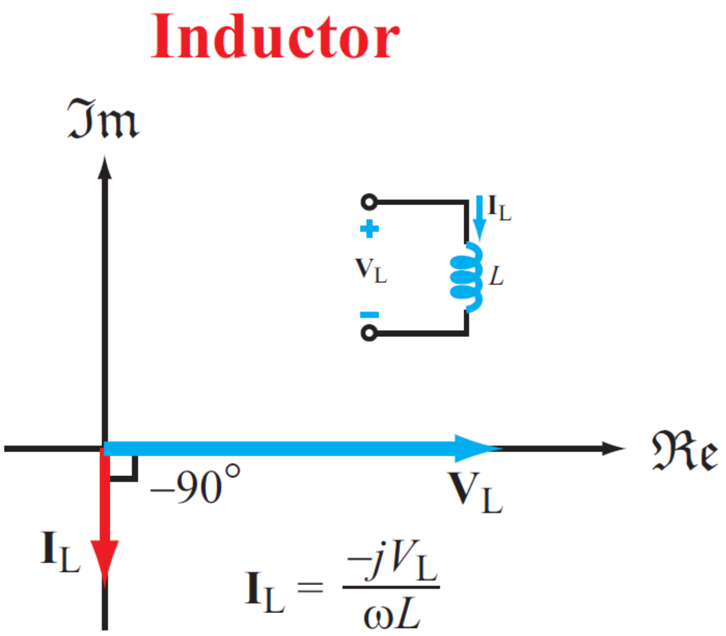
$$R_{Th} = 8.42\ \Omega,$$

$$C_{Th} = \frac{1}{1.59\omega} = 6.29\ \mu F$$

Solving using Phasor Diagrams

- The relationships between current and voltage for L and C are:

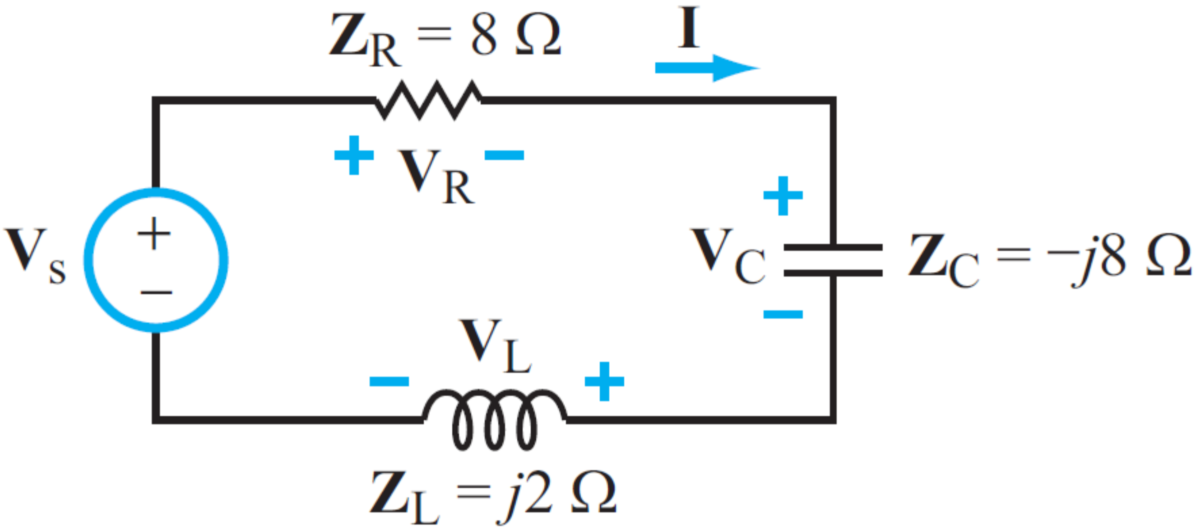




- The relationship between current and voltage for R is trivial, obviously

Solving using Phasor Diagrams

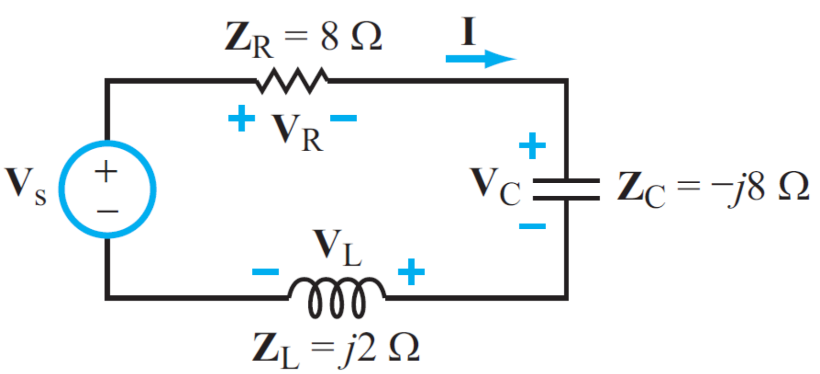
- Consider the following circuit, with Vs=20e^{j30}

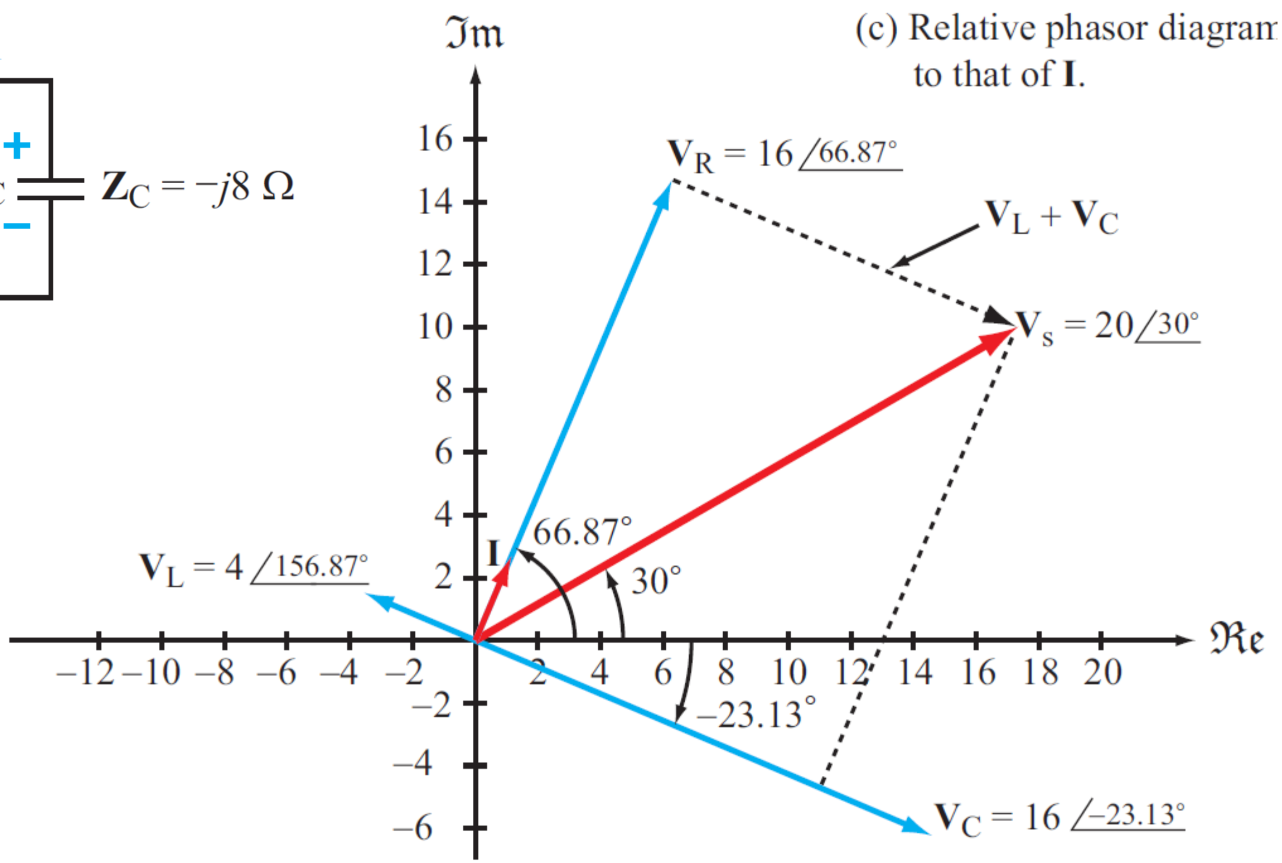


$$\mathbf{I} = \frac{\mathbf{V}_s}{R + j\omega L - \dfrac{j}{\omega C}}$$

Solving using Phasor Diagrams

- We can the find the individual voltages graphically:
  $I = 2e^{j66.87^\circ}$ A





(c) Relative phasor diagram to that of **I**.